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Closed string tachyon condensation at $c = 1$

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Abstract: The $c = 1$ matrix model, with or without a type 0 hat, has an exact quantum solution corresponding to closed string tachyon condensation along a null surface. The condensation occurs, and spacetime dissolves, at a finite retarded time on $\mathcal{I}^+$. The outgoing quantum state of tachyon fluctuations in this time-dependent background is computed using both the collective field and exact fermion pictures. Perturbative particle production induced by the moving tachyon wall is shown to be similar to that induced by a soft moving mirror. Hence, despite the fact that $\mathcal{I}^+$ for the tachyon is geodesicaly incomplete, quantum correlations in the incoming state are unitarily transmitted to the outgoing state in perturbation theory. It is also shown that, non-perturbatively, information can leak across the tachyon wall, and tachyon scattering is not unitary. Exact unitarity remains intact only in the free fermion picture.

Keywords: Tachyon Condensation, 2D Gravity, Matrix Models.

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1. Introduction

The $c = 1$ matrix model provides a non-perturbatively soluble string theory in 1+1 dimensions. However, despite a wealth of detailed data, the physical interpretation of many aspects of the theory remained unclear, especially during the first section of the story a decade ago. The hoped-for nonperturbative lessons did not fully materialize (one notable exception is [1]) and the nonperturbative consistency of the theory was questioned [2]. A cogent summary of the lessons and disappointements can be found in the conclusions of [3].

A new section of the $c = 1$ story was recently opened with the reinterpretation of the theory as an example of open-closed holography [4, 5] and as a type 0 string theory [6, 7]. As a result, the study of some non-perturbative stringy phenomena became possible. Much light was shed on the problem of unstable D-brane decay [5, 8], which can be viewed as *open* string tachyon condensation. A family of exact solutions to the theory, found in [9, 10], were shown to correspond physically to the time-dependent process of closed string tachyon condensation[1] [11]. Exact closed cosmologies were also constructed [11].

[1] We follow the standard abuse of notation in referring to the massless field of the $c = 1$ theory as a tachyon. We note that the tachyon cannot condense spontaneously in the usual vacuum, which is perturbatively stable. The process we consider involves a change of boundary conditions at infinity. Nevertheless we hope it provides a useful toy model for higher dimensional spontaneous tachyon condensation. Since the tachyon mode is in fact massless, the usual problem with defining initial state for an unstable mode does not arise.

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The study of *open* string tachyon condensation, which can be described by an exact boundary CFT [12], has led to many insights into the nonperturbative character of time-dependent string theory. Studies of *closed* string tachyon condensation have been more limited (see the review [13]). The case of localized tachyons was considered in [14]–[18] and some aspects of bulk tachyons in [19]–[26]. The main problem, of course, is that spacetime disappears altogether when a bulk tachyon condenses and hence perturbation theory breaks down. The existence of non-perturbatively exact quantum solutions at $c = 1$ provides an ideal framework to study this process.

This paper considers the exact quantum description of closed string tachyon condensation along a null hypersurface. In this process the Fermi sea — together with the spacetime described by its fluctuating surface — comes to an end at a finite retarded time t^-_{end} on $\mathcal{I}^+$. The everywhere-timelike tachyon wall — which is static in the far past and accelerates up to t^-_{end} — distorts the perturbative incoming vacuum much like a moving mirror softened at the string scale. Perturbatively, this moving mirror reflects all the information incoming from $\mathcal{I}^-$ into an outgoing excited state in the region of $\mathcal{I}^+$ *prior* to t^-_{end}.

The nonperturbative picture can be understood in the free fermion formulation. We find that nonperturbatively information/correlations can leak through the tachyon wall into the no-man's land beyond the end of time t^-_{end}. This means there is no exact unitary S-matrix for tachyon scattering. In the free fermion picture there is exact unitarity, but it involves states with no spacetime interpretation.

So, in this example, exact unitarity of the holographic dual does not imply the existence of an exactly unitary spacetime picture.

In addition to the intrinsic interest of tachyon condensation, another motivation for the present work is towards an understanding of perhaps the most tantalizing of all non-perturbative phenomena: black hole formation/evaporation, in the $c = 1$ context. One possibility is that black holes simply are not part of the theory, or cannot be made as any kind of excitation of the vacuum. Another is that they do occur but we have not yet recognized them. Adding to the mystery and frustration is the existence of an exact worldsheet CFT describing the eternal black hole [27] as well as a deformed matrix model describing the euclidean version [28]. As in the AdS/CFT correspondence, the observables of the matrix model are best thought of as living at the spacetime boundary, namely $\mathcal{I}^{\pm}$. The physical interpretation of quantities in the interior is obscured by non-local transforms and strong coupling. Hence, black hole formation might best be recognized by the appearance of Hawking radiation at $\mathcal{I}^+$. In the present work, the groundwork for the understanding of such quantum particle production is laid.

A spacetime intepretation of the free fermion variables on the past and future boundaries $\mathcal{I}^{\pm}$ involves both a bosonization and a nonlocal "leg pole" transform. The working assumption of this paper is that this map and the set of boundary observables are the same for the large processes such as tachyon condensation considered here as for the static fermi sea. This assumption leads to an apparently self-consistent picture. However, we believe that it is quite possible that there are other interesting interpretations of the boundary data on $\mathcal{I}^{\pm}$ that differ from the one employed herein perhaps by field or coordinate redefintions, although we have no specific proposal along these lines.

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This paper is organized as follows. In section 2 we review the semiclassical matrix model solution corresponding to tachyon condensation along an asymptotically null hypersurface. In the free fermion picture it corresponds to a draining of the Fermi sea, while in the spacetime picture the initially static tachyon wall accelerates up to $\mathcal{I}^+$. In section 3 the semiclassical quantum fluctuations around this solution are described using the Das-Jevicki [29] collective field formalism, together with the non-local leg pole transform [30] which turns the collective quantum fluctuations of the Fermi surface into spacetime tachyons. We find that, prior to the leg pole transform, the motion of the tachyon wall deforms the quantum vacuum like an impenetrable accelerating mirror which reaches $\mathcal{I}^+$ at the end of the world at t^-_{end}. The outgoing state of the quantum field is computed. Perturbatively, all the correlations of the incoming vacuum state of the collective field are transmitted to an excited outgoing state with support prior to t^-_{end}. The collective field formalism gives no information about the quantum state in the no-man's land after t^-_{end}. The leg pole transform leads to a similar picture for the spacetime tachyon, except that the mirror is soft rather than impenetrable and spacetime ends less abruptly. A concrete measure of the deviation of the physics on $\mathcal{I}^+$ from that of an ordinary massless field is the commutator Δ of the tachyon field with its time derivative. For normalizable fluctuations of the static Fermi surface this remains precisely a delta function, but here we find (due to the large nature of the background fluctuation) an exact expression which decays exponentially to zero after the end of the world. In section 4 we redo the collective field analysis using the exact, but less intuitive, free fermion picture. An exact quantum state[2] is given whose semiclassical limit is the null tachyon condensation. An exact computation of the commutator Δ is performed and found, to our surprise, to agree exactly with the collective field computation on $\mathcal{I}^+$. Hence, the free fermion picture corroborates the collective field analysis. Finally in section 5 nonperturbative effects are considered. It is argued that non-perturbative excitations — such as D-branes — can escape across the wall into the no-man's land and never reappear on $\mathcal{I}^+$ before the end of the world. Hence, the spacetime tachyon picture misses nonperturbative degrees of freedom in an essential manner and cannot be exactly unitary. Although there is exact unitarity in the free fermion picture, we expect there is no unitary spacetime description of the process of tachyon condensation.

We expect similar phenomena, in which both sides of the Fermi sea are drained, to occur in the type 0 matrix models. It would be interesting to consider this in detail.

We set $\alpha' = 1$ throughout the paper.

2. A moving tachyon wall solution

In this section, we will briefly review a semiclassical solution of the $c = 1$ matrix model having an interpretation as tachyon condensation along a null hypersurface [11].

As is well known, the matrix model reduces to a system of nonrelativistic, free fermions in 1 space dimension, whose potential in the double scaling limit becomes simply $-\frac{1}{2}x^2$, while the number of fermions is taken to infinity. The classical limit of this system is

[2] More precisely, since the fluctuations involved in tachyon condensation are non-normalizable in the sense of [31], this is best described by a deformed hamiltonian.

the motion of an incompressible Fermi fluid in phase space, (x, p). The static solutions of this problem are Fermi surfaces given by $x^2 - p^2 = 2\mu$. Perturbations about this static solution reproduce the perturbative behavior of the 2D Liouville string. The two branches of the hyperbola can be treated separately in perturbation theory. Nonperturbatively they interact via fermion tunneling under the potential. The spacetime interpretation of the two coupled Fermi seas together was given in [6, 7]: the symmetric and antisymmetric fluctuations of the entire Fermi sea correspond to the tachyon field and the RR flux in type 0B string theory, respectively.

Other, time dependent, solutions can be considered [9, 10, 32, 33, 11, 34], and our subject is to understand the spacetime interpretation of one such solution on the quantum level.

A particular solution of the equations of motion for the Fermi surface which we will focus on is given by[3]

$$(x + p - 2\lambda e^t)(x - p) = 2\mu\,, \tag{2.1}$$

where μ, λ are arbitrary non-negative constants. An exact quantum representation in terms of free fermions will be given in section 4. This is a *moving* hyperbola centered at $(x, p) = (\lambda e^t, \lambda e^t)$ rather than the origin. Introducing a parameter $-\infty < \sigma < \infty$ along the curve, the right branch of the solution to (2.1) may be written in the alternate useful form

$$\begin{aligned} x &= \sqrt{2\mu}\cosh\sigma + \lambda e^t\,, \\ p &= \sqrt{2\mu}\sinh\sigma + \lambda e^t\,. \end{aligned} \tag{2.2}$$

In section 4 we will give an exact quantum description in the free Fermi picture of a state whose Fermi surface obeys (2.2) in the semiclassical limit.

From (2.1), we see that in the infinite past the Fermi sea is essentially static and filled up to the energy μ below the top of the potential. It then decays away (rolls down the potential) and all fermions move out to $x = \infty$ in the infinite future. Physically, this corresponds to tachyon condensation along the asymptotically null hypersurface $t - \ln x = -\ln\lambda$ [11]. Using the dictionary of [30]–[35] in the asymptotic region of large $x = e^q$, the solution (2.1) corresponds to a tachyon field given, to leading order at large X^1, by

$$T = \hat{\mu} X^1 e^{-2X^1} + \hat{\lambda} e^{-X^1 + X^0}\,, \tag{2.3}$$

where $\hat{\lambda}$ and $\hat{\mu}$ are proportional to λ and μ.[4] Including this field leads to the closed string worldsheet interaction

$$\frac{1}{4\pi}\int d^2\,\sigma\sqrt{\hat{g}}\Big\{2X^1\hat{R} + \hat{\mu}X^1 e^{-2X^1} + \hat{\lambda}e^{-X^1+X^0}\Big\}\,. \tag{2.4}$$

The first interaction term corresponds to standard Liouville potential which is a timelike "wall" at $X^1 \sim \frac{1}{2}\ln\hat{\mu}$. The second interaction term (which is a weight (1,1) operator) is also an exponential potential wall, albeit less steep than the standard one. This wall moves with time, effectively cutting off the universe at a time-varying distance. It is located roughly along the outgoing null trajectory at $X^1 - X^0 \sim \ln\hat{\lambda}$. Hence, at large positive X^0, the wall moves outwards with essentially the speed of light. The situation is depicted in figure 1.

[3]The discussion will focus of the right branch of the hyperbola until section 4. For a type 0B solution we should include the second filled region at negative x, but for the sake of brevity we do not consider this.

[4]A divergence in the leg pole factor leads to an infinite renormalization of $\hat{\lambda}$ in terms of λ.

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Any observer moving along a timelike trajectory will eventually move into a region where the tachyon field becomes arbitrarily large. Therefore this solution is a form of closed string tachyon condensation [11].

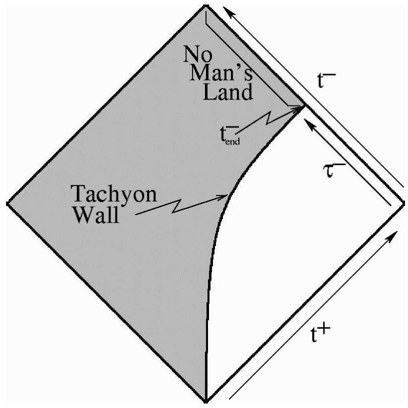

Figure 1: Penrose diagram for lightlike tachyon condensation. The timelike curve ending on $\mathcal{I}^+$ is the tachyon wall. There are no perturbative degrees of freedom propagating in the shaded region.

3. The collective field picture

The previous section used the dictionary developed in [35] to obtain the spacetime interpretation of a moving Fermi surface. This dictionary is valid only for small deviations from the static Fermi surface, and breaks down at late times for our solution when the deviations become large. In this section we will use the (in this respect) more general and powerful Das-Jevicki collective field formalism [29] (see also the recent discussion [36]). The goal will be to determine the outgoing quantum state of the tachyon field.

3.1 Classical action for small fluctuations

We are interested in expressing the effective action for the collective fermion field. The action, in "fermion coordinates" is [29]

$$\int dtdx\left[\frac{Z^2}{2\pi\varphi}-\frac{\varphi^3}{6\pi}+\frac{1}{\pi}\left(\frac{1}{2}x^2-\mu\right)\varphi\right], \tag{3.1}$$

where φ is defined in terms of the upper and lower Fermi surfaces $p_\pm$ by

$$\varphi(x,t)\equiv\frac{1}{2}(p_+(x,t)-p_-(x,t)), \tag{3.2}$$

while Z is defined by

$$Z(x,t)\equiv\int^x dx'\partial_t\varphi(x'). \tag{3.3}$$

The collective field η describes the difference between a fluctuating Fermi surface φ and a background solution φ_0:

$$\varphi=\varphi_0+\sqrt{\pi}\partial_x\eta. \tag{3.4}$$

From (3.1), we obtain the action for η

$$\int dtdx\left[\frac{1}{2\pi}\frac{(Z_0+\sqrt{\pi}\partial_t\eta)^2}{\varphi_0+\sqrt{\pi}\partial_x\eta}-\frac{1}{6\pi}(\varphi_0+\sqrt{\pi}\partial_x\eta)^3+\frac{1}{\pi}\left(\frac{1}{2}x^2-\mu\right)(\varphi_0+\sqrt{\pi}\partial_x\eta)\right]. \tag{3.5}$$

In the case of present interest (2.1) we have

$$\varphi_0=\sqrt{(x-\lambda e^t)^2-2\mu}, \qquad Z_0=-\lambda e^t\varphi_0. \tag{3.6}$$

The action (3.5) contains the quadratic piece

$$S_{(2)}=\frac{1}{2}\int\frac{dt\ dx}{\varphi_0}\left[(\partial_t\eta)^2-2\frac{Z_0}{\varphi_0}(\partial_t\eta\partial_x\eta)-(\varphi_0^2-\frac{Z_0^2}{\varphi_0^2})\left(\partial_x\eta\right)^2\right], \tag{3.7}$$

and a leading interaction piece

$$S_{(3)} = \int dt\; dx\; \varphi_0^{-2} o(\eta^3)\,. \tag{3.8}$$

It is sometimes convenient to introduce light cone fermion coordinates

$$t^{\pm} = t \pm \ln x = t \pm q\,, \tag{3.9}$$

in which the quadratic action (3.7) for the solution (3.6) becomes

$$S_2 = \int \frac{dt^+ dt^-}{\sqrt{(1-\lambda e^{t^-})^2 - 2\mu e^{t^- - t^+}}} \left[(1-\lambda e^{t^-})\partial_+\eta\partial_-\eta + \lambda e^{t^-}(\partial_+\eta)^2 + \frac{\mu}{2} e^{t^- - t^+}(\partial_+\eta - \partial_-\eta)^2 \right]. \tag{3.10}$$

Note that the last μ-dependent term vanishes both on $\mathcal{I}^+$ ($t^+ \to +\infty,\;\; t^-$ finite) and $\mathcal{I}^-$ ($t^- \to -\infty,\;\; t^+$ finite). However, it plays a crucial role in keeping the tachyons out of the strong coupling region. On $\mathcal{I}^-$ the λ terms also vanish and η is a canonical free boson. The λ terms do not vanish on $\mathcal{I}^+$, where it represents the effects of the background null tachyon.

Following [37], it is useful to define the "Alexandrov coordinates" $\tau^{\pm} = \tau \pm \sigma$, with σ defined by (2.2) and $\tau = t$. Specifically,

$$x = \sqrt{2\mu}\cosh(\sigma) + \lambda e^{\tau}\,. \tag{3.11}$$

This implies

$$t^{\pm} = \tau^{\pm} \pm \ln\left(\sqrt{\frac{\mu}{2}} + \lambda e^{\tau^-} + \sqrt{\frac{\mu}{2}} e^{\tau^- - \tau^+}\right). \tag{3.12}$$

It is then easy to show that the partial derivatives of the Alexandrov coordinates with respect to the original fermion coordinates can be written as

$$\partial_x \sigma = \frac{1}{\varphi_0}\,, \qquad \partial_t \sigma = \frac{Z_0}{\varphi_0^2}\,, \qquad \partial_x \tau = 0\,, \qquad \partial_t \tau = 1\,. \tag{3.13}$$

Using the above set of partial derivatives, it is not hard to show that the action given in (3.5) becomes in the Alexandrov coordinates

$$\begin{aligned} \int d\tau d\sigma \Bigg[& \frac{1}{2}((\partial_\tau\eta)^2 - (\partial_\sigma\eta)^2) - \frac{\sqrt{\pi}}{6\varphi_0^2}(3(\partial_\tau\eta)^2(\partial_\sigma\eta) + (\partial_\sigma\eta)^3) + \\ & + \sum_{n=2}^{\infty} \frac{(-1)^n}{2}(\partial_\tau\eta)^2 \left(\frac{\sqrt{\pi}(\partial_\sigma\eta)}{\varphi_0^2}\right)^n \Bigg]. \end{aligned} \tag{3.14}$$

In the (τ,σ) coordinates, $\varphi_0 = \sqrt{2\mu}\sinh(\sigma)$, so the above action is static. In fact, all effects due to $\lambda \neq 0$ have disappeared, and (3.14) is identical to the effective action for the pure Liouville background $\lambda = 0$. In particular, the coupling varies exponentially with the coordinate σ: $g \sim \exp(-2\sigma)$, and is independent of time τ. The quadratic term for η is a canonical free field. In addition there is a reflecting boundary condition along the timelike line $\tau^+ = \tau^-$ [29],

$$[\partial_+\eta - \partial_-\eta]_{\tau^+ = \tau^-} = 0\,. \tag{3.15}$$

This line parameterizes the end of the Fermi sea, where incoming fluctuations round the bend and turn into outgoing fluctuations. Now we see the advantage of these coordinates. The general solution of the linearized wave equation following from (3.5) can now easily be found by transforming to Alexandrov coordinates where they are simply plane waves reflected off the origin.

3.2 The leg pole transform

The S-matrix for η scattering agrees with that of spacetime tachyon scattering only after performing the nonlocal leg pole transform [38, 39, 30]. We will now use the leg pole transform to convert the effective variable η into a dressed spacetime tachyon S. The leg pole transform is only known in the asymptotic regimes $\mathcal{I}^{\pm}$, and we will focus on $\mathcal{I}^{+}$ (since behavior at $\mathcal{I}^{-}$ is standard). Outgoing waves in this region obey $\eta(\sigma,\tau) = \eta(\tau-\sigma)$. On $\mathcal{I}^{+}$, η will be written as a function of *one* variable, and we will always mean it as a function of $\tau^{-} = \tau - \sigma$. Comparing the definition of η in (3.4) with the definition of $\bar{S}$ in [30],

$$\varphi = \frac{1}{2}(p_{+} - p_{-}) = e^{q} + \sqrt{\pi} e^{-q} \partial_q \bar{S}(q,t)\,, \qquad q \equiv \ln x\,, \tag{3.16}$$

we see that

$$\bar{S} = \eta + \pi^{-1/2} \int (x - \varphi_0)\,. \tag{3.17}$$

A point on $\mathcal{I}^{+}$ labelled by $t^{-} = t - q$ corresponds in the Alexandrov coordinates to

$$\tau^{-} = t^{-} + \ln\left(\sqrt{\frac{\mu}{2}}\right) - \ln\left(1 - \lambda e^{t^{-}}\right). \tag{3.18}$$

Notice that τ^{-} is a function of t^{-} (and not t^{+}) only in this limit. The leg pole transform of $\eta(\tau^{-})$ reads therefore[5]

$$\begin{aligned} S(t^{-}) &= \int dv^{-} K(v^{-}) \eta\left(t^{-} - v^{-} + \ln\left(\sqrt{\frac{\mu}{2}}\right) - \ln(1 - \lambda e^{t^{-} - v^{-}})\right) \\ &= \int dw^{-} \frac{K\left(w^{-} + \ln(1 + \lambda e^{t^{-} - w^{-}})\right)}{1 + \lambda e^{t^{-} - w^{-}}} \eta\left(t^{-} - w^{-} + \ln\left(\sqrt{\frac{\mu}{2}}\right)\right). \end{aligned} \tag{3.19}$$

It is not difficult to convince oneself that for $\lambda e^{t^{-} - w^{-}} \ll 1$,

$$\frac{K\left(w^{-} + \ln(1 + \lambda e^{t^{-} - w^{-}})\right)}{1 + \lambda e^{t^{-} - w^{-}}} \approx K(w^{-})\,. \tag{3.20}$$

Hence, the tachyon $S(t^{-})$ is appreciably affected by $\lambda \neq 0$ only in the region where $t^{-} - w^{-} > -\ln \lambda$.

The kernel $K(v)$ is defined as [30]

$$\begin{aligned} K(v) &= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega v} \left(\frac{\pi}{2}\right)^{-i\omega/4} \frac{\Gamma(-i\omega)}{\Gamma(i\omega)} = -\frac{z}{2} J_1(z) = \frac{d}{dv} J_0(z)\,, \\ z(v) &\equiv 2\left(\frac{2}{\pi}\right)^{1/8} e^{v/2}\,. \end{aligned} \tag{3.21}$$

[5] We note that the leg pole transform is defined in the $t \pm q$ coordinates on $\mathcal{I}^{+}$. This is our working definition of the spacetime theory. In principle there may be other possibilities.

The asymptotic behavior is

$$K(v) \sim e^{v}\,, \qquad v \to -\infty \tag{3.22}$$

and

$$K(v) \sim e^{v/4}\cos\left(z+\frac{\pi}{4}\right), \qquad v \to +\infty\,. \tag{3.23}$$

K decays exponentially for $v \to -\infty$ and grows while oscillating wildly for $v \to +\infty$.

We are interested in finding the leg pole transform $S_k(t^-)$ of an outgoing mode of η of momentum k

$$\eta_k(\tau^-) = \frac{e^{-ik\tau^-}}{\sqrt{2|k|}}\,. \tag{3.24}$$

Let's consider first the region $t^- < -\ln\lambda$, where (3.20) is valid. Thus, the main contribution to $S_k(t^-)$ comes from region around $w^- = 0$. As a result, $S_k(t^-)$ is essentially $\eta_k(\tau^- = t^- + \ln(\sqrt{\mu/2}))$ with some smearing.

In order to study the region $t^- > -\ln\lambda$, it turns out to be more convenient to rewrite (3.19) (integrating by parts) as

$$S(t^-) = \int dv^- J_0\Big(z(v^- + \ln(1+\lambda e^{t^- - v^-}))\Big)\partial\eta\left(t^- - v^- + \ln\left(\sqrt{\frac{\mu}{2}}\right)\right). \tag{3.25}$$

Using the integral representation of J_0

$$J_0(z(y)) = \int \frac{d\omega}{2\pi}\left(\frac{\pi}{2}\right)^{-i\omega/4}\frac{\Gamma(-i\omega)}{\Gamma(i\omega+1)}e^{i\omega y}\,, \tag{3.26}$$

together with a contour prescription where the integral over the real line passes *over* the pole at $\omega = 0$, we obtain that the leg pole transform of η_k is

$$S_k(t^-) = \int \frac{d\omega dv^-}{2\pi}\left(\frac{\pi}{2}\right)^{-i\omega/4}\frac{\Gamma(-i\omega)}{\Gamma(i\omega+1)}\,e^{i\omega v^-}\,(1+\lambda e^{t^- - v^-})^{i\omega}\left(\frac{-ike^{-ik(t^- - v^- + \ln(\sqrt{\mu/2}))}}{\sqrt{2|k|}}\right). \tag{3.27}$$

It is easiest to evaluate the integral over v^- first, after taking ω and k slightly off the real axis to regularize it. This way, we obtain

$$S_k(t^-) = \int \frac{d\omega}{2\pi}\left(\frac{\pi}{2}\right)^{-i\omega/4}\lambda^{i(\omega+k)}\frac{\Gamma(-ik-i\omega)\,\Gamma(1+ik)}{\Gamma(i\omega+1)}\frac{e^{i\omega(t^-)}}{\sqrt{2|k|}}\left(\sqrt{\frac{\mu}{2}}\right)^{-ik}. \tag{3.28}$$

This integral can be evaluated by closing the contour in the lower half plane, and summing over all the poles of $\Gamma(ik - i\omega)$ (including the pole at $\omega = k$, according to the contour prescription arising from regularization of the v^- integral). The resulting series sums to

$$S_k(t^-) = \frac{1}{\sqrt{2|k|}}\left(\frac{\sqrt{2\mu}z}{\lambda}\right)^{-ik}J_{-ik}(z)\Gamma(1+ik)\,, \tag{3.29}$$

where we have defined

$$z \equiv 2\left(\frac{2}{\pi}\right)^{1/8}(\lambda e^{t^-})^{1/2}\,. \tag{3.30}$$

The dressed tachyon $S(q,t)$ is related to the undressed tachyon field by $T = e^{\Phi} S$. However, we are actually more interested in the field S itself, which is the properly normalized massless excitation of the tachyon field.

While it is not possible to extract the dilaton from the data presented here, the asymptotic form of the dilaton should be $\Phi \sim -2q$, consistent with the following observation: up to the smearing present in the transform (3.19), and for $t^- < -\ln\lambda$, we have $\sigma = q - \ln(\sqrt{2\mu})$. The action (3.14) has a coupling constant $g \sim \phi_0^{-2} \sim e^{-2\sigma}$. This suggest that the dilaton is linear in the region in which perturbations propagate freely (i.e. for $t^- \ll -\ln(\lambda)$).

3.3 The quantum vacuum for the collective field

Action (3.10) is time-dependent. However, in the far past the background is static and there is a natural incoming vacuum state. To determine the outgoing state as usual we must solve the time-dependent wave equation. In this subsection we discuss the situation in the fermion light cone coordinates $t^{\pm} = t \pm q$ and in the next subsection we consider the effects of the leg pole transform.

The exact linearized wave equation for η is obtained by varying (3.10):

$$\left((1-\lambda e^{t^-})\partial_+\partial_- + \lambda e^{t^-}\partial_+\partial_+ + \frac{\mu}{2}e^{t^- - t^+}((\partial_+ - \partial_-)^2 - (\partial_+ - \partial_-))\right)\eta = 0\,. \qquad (3.31)$$

In the far past, on $\mathcal{I}^-$, the equation reduces simply to

$$\partial_+\partial_-\eta = 0\,, \qquad \text{on } \mathcal{I}^-\,. \qquad (3.32)$$

In the far future we have

$$\left(\partial_+\partial_- + \frac{\lambda e^{t^-}}{(1-\lambda e^{t^-})}\partial_+\partial_+\right)\eta + = 0\,, \qquad \text{on } \mathcal{I}^+\,. \qquad (3.33)$$

Notice that outgoing plane waves, which are functions of t^- only, remain solutions of (3.33) on $\mathcal{I}^+$ for nonzero λ. We seek solutions of the wave equation which are purely positive frequency on $\mathcal{I}^-$

$$u^{\rm in}_\omega(t^+,t^-) \to \frac{e^{-i\omega t^+}}{\sqrt{2\omega}}\,, \qquad \text{for } t^- \to -\infty, \omega > 0\,. \qquad (3.34)$$

Such solutions are orthonormal

$$\langle u^{\rm in}_{\omega'}|u^{\rm in}_{\omega}\rangle \equiv i\int_\Sigma \frac{d\Sigma^\mu}{2\pi} u^{in*}_\omega \overleftrightarrow{\partial_\mu} u^{\rm in}_{\omega'} = \delta(\omega-\omega')\,, \qquad (3.35)$$

where Σ is any complete spacelike or null slice and $d\Sigma^\mu$ the normal volume element with respect to the metric appearing in (3.10). The modes are also complete on $\mathcal{I}^-$

$$2i\int_0^\infty \frac{d\omega}{2\pi}(u^{in*}_\omega(t'^+)\partial_+ u^{\rm in}_\omega(t^+) - \partial_+ u^{in*}_\omega(t^+)u^{\rm in}_\omega(t'^+)) = \delta(t^+ - t'^+)\,. \qquad (3.36)$$

We then expand

$$\eta(t^+,t^-) = \int_0^\infty d\omega(a^{in\dagger}_\omega u^{in*}_\omega(t^+,t^-) + a^{\rm in}_\omega u^{\rm in}_\omega(t^+,t^-))\,, \qquad (3.37)$$

and the quantum vacuum state is defined by

$$a_\omega^{\rm in}|0_{\rm in}\rangle = 0\,. \tag{3.38}$$

The full solutions of the wave equation can be written

$$u_\omega^{\rm in}(t^+,t^-) = \frac{e^{-i\omega\tau^+}}{\sqrt{2\omega}} + \frac{e^{-i\omega\tau^-}}{\sqrt{2\omega}}\,, \tag{3.39}$$

where the Alexandrov coordinates $\tau^\pm(t^+,t^-)$ are given in (3.12). These solutions obey reflecting boundary condition (3.15) along the timelike line $\tau^+ = \tau^-$. In the $t^\pm$ fermion coordinates the reflecting boundary conditions are at

$$t_R^+(t^-) = t^- + \ln 2\mu - 2\ln(1-\lambda e^{t^-})\,. \tag{3.40}$$

This trajectory is everywhere timelike but accelerates and reaches $\mathcal{I}^+$ at the end of the world

$$t^-_{\rm end} = -\ln\lambda\,. \tag{3.41}$$

The behavior of the tachyon vacuum is similar to the vacuum in the presence of a mirror moving along the trajectory (3.40). This latter problem is discussed in [40].

Condition (3.38) defines the linearized quantum state but we are particulary interested in the behavior of outgoing modes on $\mathcal{I}^+$, where both $t^+\to\infty$ and $\tau^+\to\infty$. In this region the outgoing part of the in modes (3.39) is

$$\begin{aligned} &t^+\to\infty\,, \qquad u_\omega^{\rm in} \to \left[\sqrt{\frac{2}{\mu}}(e^{-t^-}-\lambda)\right]^{i\omega}\,, \qquad t^- < t^-_{\rm end}\,, \\ &u_\omega^{\rm in} \to 0\,, \qquad t^- > t^-_{\rm end}\,, \end{aligned} \tag{3.42}$$

while the natural basis of positive frequency out modes is

$$u_\omega^{\rm out} = \frac{e^{-i\omega t^-}}{\sqrt{2\omega}}\,. \tag{3.43}$$

Since (3.42) and (3.43) are not the same, an observer near $\mathcal{I}^+$ moving along a timelike trajectory of constant $t^+ - t^-$ will detect an outgoing particle flux prior to the end of the world. An observer accelerating along constant $\tau^+ - \tau^-$ will detect nothing.

Before the end of the world the outgoing quantum state can be expressed as an excitation of the out vacuum. Define the Bogolubov coefficients

$$\alpha_{\omega\omega'} = \langle u_{\omega'}^{\rm out}|u_\omega^{\rm in}\rangle\,, \qquad \beta_{\omega\omega'} = -\langle u_{\omega'}^{\rm out *}|u_\omega^{\rm in}\rangle\,, \tag{3.44}$$

and the out creation operators

$$a_\omega^{\rm out\dagger} = -i\int_{-\infty}^{t^-_{\rm end}} \frac{dt^-}{2\pi} u_\omega^{\rm out}(t^-)\,\overleftrightarrow{\partial}_-\,\eta(t^-)\,, \tag{3.45}$$

which create plane waves with support prior to $t^-_{\rm end}$. Then the outgoing quantum state can be written prior to the end of the world as

$$e^{\frac{1}{2}\int_0^\infty d\omega d\omega' a^{\rm out\dagger}_\omega (\alpha^{-1}\beta)^*_{\omega\omega'} a^{\rm out\dagger}_{\omega'}}|0_{\rm out}\rangle\,, \tag{3.46}$$

where $a^{\rm out}_\omega|0_{\rm out}\rangle = 0$.

We wish to emphasize the obvious fact that we have not determined the quantum state on all of $\mathcal{I}^+$. Although they remain orthonormal, the in modes $(u^{\rm in}_\omega, u^{in*}_\omega)$ in (3.39) do not evolve to a complete set of modes on $\mathcal{I}^+$, and in particular vanish in no-man's land. Later on we shall interpret this as the disappearance of spacetime in that region.

The outgoing energy density is given (before the world ends) on $\mathcal{I}^+$ by the Schwarzian from Alexandrov to light cone fermion coordinates:

$$\begin{aligned} T_{--}(t^-) &= -\frac{1}{12}\left(\frac{\partial\tau^-}{\partial t^-}\right)^{3/2}\frac{\partial^2}{\partial\tau^{-2}}\left(\frac{\partial\tau^-}{\partial t^-}\right)^{1/2} \\ &= -\frac{\lambda e^{t^-}(2-\lambda e^{t^-})}{48(1-\lambda e^{t^-})^2}\,. \end{aligned} \tag{3.47}$$

Note that the energy density (3.47) is negative and diverges at the critical retarded time (3.41). The form of the stress energy (3.47) is similar to that at the edge of the Rindler wedge in the Rindler vacuum, or at the horizon of a black hole in the Boulware vacuum.[6] It can be viewed as a Casimir energy arising from the reflecting boundary condition imposed at the tachyon wall. This behavior is exactly what one expects viewing the moving tachyon wall as a moving mirror. In the next subsection we shall see from the leg pole transform that the spacetime picture is similar, except that this impenetrable boundary is softened at the string scale.

In closing this section we note the preceding analysis breaks down along the reflecting boundary (3.40) because, among other reasons, the Casimir energy density is diverging there. The best way to overcome this problem is to work in the free fermion picture, to which we turn in section 4.

3.4 Spacetime particle production

Now we leg pole transform to the spacetime picture. This is easily accomplished by simply replacing the expression (3.42) for the in modes $u^{\rm in}_\omega(t^-)$ in the out region with their leg pole transforms $S_\omega(t^-)$ in equation (3.29). While the modes in (3.42) terminate abruptly at the end of the world, the S_ω modes decay exponentially after the end of the world. Hence, spacetime dissolves away on a time period of order the string time. Nevertheless prior to this cataclysmic event, at the level of free fields all the information about the incoming quantum state is transmitted to $\mathcal{I}^+$.

Viewing the tachyon wall in spacetime as a soft moving mirror, this seems a natural conclusion. However, from another point of view it seems strange. The modes (3.39) vanish identically in the region above the null tachyon wall, since the Alexandrov coordinates

[6] This raises the very interesting question of whether or not there exists an alternate vacuum which is the analog of the Hartle-Hawking vacuum for this problem and has a finite stress energy tensor.

simply do not extend into that region. Why can't they tunnel through the wall with exponentially small amplitudes? Why can't quantum correlations be lost to the no-man's land after the end of the world? We shall argue below in section 5 that, while neither of these phenomena occur in the perturbative collective field description, they in fact can be seen to occur in the non-perturbative free fermion formulation.

It is useful to give a quantitative measure of the rate at which spacetime disappears. To this end, we will consider the expectation value of the following commutator

$$\Delta(q,q';t) = \langle [\partial_t S(t,q), S(t,q')] \rangle \,, \tag{3.48}$$

where the expectation value is taken in the state $|0_{\rm in}\rangle$. In field theory we always have

$$\Delta(q,q';t) = \delta(q-q') \,, \tag{3.49}$$

independently of the quantum state. It is easy to see that, for normalizable perturbations of the static incoming Fermi sea, (3.49) remains exactly valid, and the notion of a complete $\mathcal{I}^+$ remains intact. In fact, if we drain the Fermi sea for a very long time, but then refill it, (3.49) is still maintained everywhere on $\mathcal{I}^+$. In contrast we shall see momentarily that tachyon condensation causes Δ to approach zero in the far future of $\mathcal{I}^+$.

On $\mathcal{I}^+$, Δ has a finite limit and so cannot be a function of (t^+, t'^+). It may be written in terms of the S_k modes as

$$\Delta(t^-, t'^-) = i \int_0^\infty \frac{dk}{\pi} (S_k(t^-)\partial S_k^*(t'^-) - S_k^*(t^-)\partial S_k(t'^-)) \,. \tag{3.50}$$

Since this integral is hard to deal with, we will compute this object in a different way: as the double leg pole transform of the η commutator. This gives

$$\begin{aligned}\Delta(t^-, t'^-) = & \int dv^- dv'^- \frac{K\Big(v^- + \ln(1+\lambda e^{t^- - v^-})\Big)}{1+\lambda e^{t^- - v^-}} \frac{K\Big(v'^- + \ln(1+\lambda e^{t'^- - v'^-})\Big)}{1+\lambda e^{t'^- - v'^-}} \times \\ & \times \left\langle \left[\partial_t \eta \left(t^- - v^- + \ln\left(\sqrt{\frac{\mu}{2}}\right)\right), \eta\left(t'^- - v'^- + \ln\left(\sqrt{\frac{\mu}{2}}\right)\right)\right]\right\rangle . \end{aligned} \tag{3.51}$$

Since η has a standard kinetic term (3.14) in (τ, σ) coordinates, we have

$$\langle [\partial_\tau \eta(\tau^-), \eta(\tau'^-)] \rangle = \delta(\tau^- - \tau'^-) \,. \tag{3.52}$$

After a change of variables, the double integral can be written as

$$\begin{aligned}\Delta(t^-, t'^-) &= \int_{t^- + \ln\lambda}^\infty dv^- \int_{t'^- + \ln\lambda}^\infty dv'^- K(v^-)K(v'^-)\delta(v^- - t^- - v'^- + t'^-)(1-\lambda e^{t^- - v^-}) \\ &= \int_{t^- + \ln\lambda}^\infty dv^- \; (1-\lambda e^{t^- - v^-}) \; K(v^-)K(v^- - t^- + t'^-) \,. \end{aligned} \tag{3.53}$$

For both t^- and t'^- large and negative, the above expression is well approximated by

$$\int_{-\infty}^\infty dv^- \; K(v^-)K(v^- - t^- + t'^-) = \delta(t'^- - t^-) \,. \tag{3.54}$$

Thus, the commutator (3.48) reduces for negative t^- to its expected form. For t^- in no-man's land, however, it is quite different, approaching zero as t^- grows, thus signifying the disappearance of spacetime degrees of freedom.

An informative expression can be given for the integral of Δ over $\mathcal{I}^+$ with respect to one of its arguments. We find

$$\int dt'^- \Delta(t'^-, t^-) = \int_{t^-}^{\infty} dv^- (1 - \lambda e^{t^- - v^-}) K(v^-) \,. \tag{3.55}$$

This goes to the field theory answer $\int \Delta = 1$ in the far past, but again approaches zero in the far future.

The discussion of this subsection employed the collective field approximation. Δ can in fact be computed exactly (including tunneling effects) using the fermion picture. Surprisingly (at least to us), we will see in the next section that the results (3.53) and (3.55) are exact.

4. The fermion picture

The above analysis was performed in the collective field description of the underlying fermion degrees of freedom. In order to understand the nonperturbative issues, as well as the potential breakdown of the semiclassical picture at the tachyon wall, we must go back to the exact quantum fermion description. A useful reference, whose notation we largely follow, is [41].

Our first task is to write down an exact *static* quantum state which we denote μ_R, in which the Fermi sea is filled up to the level $-\mu$ on the right hand side of the barrier, but is as empty as possible on the left side of the barrier. Because of the exponentially small tunneling, there are no energy eigenstates with no fermions whatsoever on the left. A complete basis of orthonormal hamitonian eigenstates obeying[7]

$$HW_\nu^{R,L} = i\partial_t W_\nu^{R,L} = -\nu W_\nu^{R,L} \tag{4.1}$$

are provided by the functions

$$W_\nu^R(x,t) = W(\nu, x) e^{i\nu t} \,, \qquad W_\nu^L(x,t) = (W(\nu, -x) + r(\nu) W(\nu, x)) e^{i\nu t} \,, \tag{4.2}$$

where $r(\nu)$ (whose explicit form shall not be needed) vanishes exponentially at large ν and the Whittaker functions $W(\nu, x)$ obey [41]

$$W(\nu, x) \sim \frac{1}{(2\pi x \sqrt{1 + e^{2\pi\nu}})} \left(\sqrt{1 + e^{2\pi\nu}} - e^{\pi\nu}\right)^{-1/2} \sin\left(\frac{x^2}{4} - \nu \ln x + \Phi(\nu)\right) \tag{4.3}$$

for $x \gg \nu$ and

$$W(\nu, x) \sim \frac{1}{(2\pi x \sqrt{1 + e^{2\pi\nu}})} \left(\sqrt{1 + e^{2\pi\nu}} - e^{\pi\nu}\right)^{1/2} \cos\left(\frac{x^2}{4} - \nu \ln x + \Phi(\nu)\right) \tag{4.4}$$

[7]Notice that ν is the energy below the top of the potential.

for $|x| \gg \nu$ and x negative (for a definition of the phase $\Phi(\nu)$, see [41]). Hence, the mode W^R (W^L) is supported largely on the right (left) side of the barrier.

We now expand the Fermi field in terms of creation and annihilation operators as

$$\Psi(x,t) = \int_{-\infty}^{\mu} d\nu a_\nu^R W_\nu^R(x,t) + \int_{\mu}^{\infty} d\nu b_\nu^{R\dagger} W_\nu^R(x,t) + \int_{-\infty}^{\infty} d\nu a_\nu^L W_\nu^L(x,t)\,. \tag{4.5}$$

The state $|\mu_R\rangle$, corresponding to filling the right side of the barrier up to energy $-\mu$, is defined by

$$a_\nu^R|\mu_R\rangle = b_\nu^R|\mu_R\rangle = a_\nu^L|\mu_R\rangle = 0\,. \tag{4.6}$$

Later on we shall need the expression for the eigenvalue density, $\rho(x)$, defined as

$$\rho(x) \equiv \langle\mu_R|\Psi^\dagger\Psi(x)|\mu_R\rangle = \int_{\mu}^{\infty} d\nu |W(\nu,x)|^2\,. \tag{4.7}$$

The asymptotic behavior of $\rho(x)$ is $\rho(x) \sim x$ for large positive x and $\rho(x) \sim \sin(x^2/4)/(x\ln(x))$ for large negative x.

We might also try to write down a coherent quantum state of the fermion theory corresponding to the classical motion of the Fermi sea. However, the difference between the quantum state describing the original static Fermi sea and the state describing the draining Fermi sea is too large to be described as a state in the Hilbert space of the original theory [31]. In the language of [31], it involves non-normalizable modes. Hence, it is associated to a new hamiltonian rather than to a semiclassical quantum state in the theory governed by the old hamiltonian.

The classical piece is given in fermion language in equation (2.1). The surface in phase space (x,p) is governed by a flow induced by a hamiltonian $(p^2 - x^2)/2$. Defining new phase space coordinates $(y,p_y) \equiv (x - \lambda e^t, p - \lambda e^t)$, we notice that these have the equations of motion $\dot{y} = p_y$, $\dot{p}_y = y$ and that in these coordinates the system is governed by a hamiltonian $(p_y^2 - y^2)/2$ — while the Fermi surface of interest is given by $(y^2 - p_y^2) = 2\mu$. We know the quantum theory corresponding to this classical limit — it is simply the theory of free fermions with potential $-y^2/2$, with all states up to μ below the top filled. In terms of the original (x,p) coordinates the time-dependent hamiltonian is

$$H_\lambda = \frac{1}{2}(p^2 - x^2) - \lambda e^t(p - x)\,. \tag{4.8}$$

Fermion correlators in the theory defined by (4.8) are easily computed in terms of the $\lambda = 0$ correlators by transforming to the (y,p_y) phase space. For example,

$$\langle\Psi^\dagger(x,t)\Psi(x',t')\rangle_\lambda \equiv \langle\Psi^\dagger(y+\lambda e^t,t)\Psi(y'+\lambda e^{t'},t')\rangle_\lambda = \langle\Psi^\dagger(y,t)\Psi(y',t')\rangle\,, \tag{4.9}$$

where

$$y(x,t) = x - \lambda e^t\,. \tag{4.10}$$

$\langle\ \cdot\ \rangle$ and $\langle\ \cdot\ \rangle_\lambda$ denote expectation values in the theory on the static background, and the time-dependent background, respectively.

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Of particular interest are correlators of fermion bilinears which determine the evolution of the shape of the Fermi sea. We have

$$\langle \Psi^\dagger\Psi(x,t)\Psi^\dagger\Psi(x',t')\rangle_\lambda \equiv \langle \Psi^\dagger\Psi(y,t)\Psi^\dagger\Psi(y',t')\rangle\,. \tag{4.11}$$

This determines the two-point function of the collective variable (eigenvalue density)

$$\varphi_0 + \sqrt{\pi}\partial_x\eta \sim \Psi^\dagger\Psi\,. \tag{4.12}$$

The values of such correlators on $\mathcal{I}^+$ determine the outgoing quantum state on $\mathcal{I}^+$. It is easy to see that the exact relation (4.11) confirms, in fermion language, our perturbative computations using the collective variable η. That is, the correlators of the previous section are the semiclassical limit of the exact correlators. Hence, the conclusions of the previous section are unaffected by the breakdown of perturbation theory at the tachyon wall.

To be very concrete about this let us compute the exact expression for the equal time commutator Δ in (3.48). We begin with

$$\langle [i\partial_t(\Psi^\dagger\Psi(x,t)), \Psi^\dagger\Psi(x',t)]\rangle_\lambda\,. \tag{4.13}$$

Define

$$G(x,x';t) \equiv \langle \Psi^\dagger(x',t)\Psi(x,t))\rangle_\lambda\,. \tag{4.14}$$

Then, it can be shown using $H\Psi = i\partial_t\Psi$ that

$$\langle [i\partial_t(\Psi^\dagger\Psi(x,t)), \Psi^\dagger\Psi(x',t)]\rangle_\lambda = \mathrm{Re}\left(\partial_x^2 G(x,x';t)\delta(x-x') - G(x,x';t)\partial_x^2\delta(x-x')\right)\,. \tag{4.15}$$

By integrating this against a test function, and using the fact that $\mathrm{Re}(G(x,x';t)) = \mathrm{Re}(G(x',x;t))$, we get that

$$\langle [i\partial_t(\Psi^\dagger\Psi(x,t)), \Psi^\dagger\Psi(x',t)]\rangle_\lambda = -\partial_x(\rho(x,t)\partial_x\delta(x-x'))\,. \tag{4.16}$$

The collective field interpretation of this quantity is

$$\langle [i\partial_t(\Psi^\dagger\Psi(x,t)), \Psi^\dagger\Psi(x',t)]\rangle_\lambda = \partial_x\partial_{x'}\langle[\dot\eta(x,t),\eta(x',t)]\rangle = \partial_x\partial_{x'}\delta(x-x')\,. \tag{4.17}$$

We now can integrate with respect to both x and x', to obtain a fermionic expression which should generalize the δ-function:

$$-\int^{x'} dy' \int^{x} dy\partial_y(\rho(y,t)\partial_y\delta(y-y')) = \rho(x,t)\delta(x-x') = \rho(x-\lambda e^t)\delta(x-x')\,. \tag{4.18}$$

This expression, when leg pole transformed, gives the exact expression for Δ. In order to compute the leg pole transform, we use the $t^\pm$ coordinates on $\mathcal{I}^+$: $2t = t^+ + t^- = 2t' = t'^+ + t'^-$ and $x = e^{(t^+ - t^-)/2}$, $x' = e^{(t'^+ - t'^-)/2}$, for $t^+ \to \infty$. We get then

$$\rho(x-\lambda e^t)\delta(x-x') = e^{(t^- - t^+)/2}\rho(e^{(t^+ - t^-)/2}(1-\lambda e^{t^-}))\delta(t'^- - t^-)\,. \tag{4.19}$$

The presence of the $e^{t^+/2}$ factor in the argument of ρ means that only the asymptotic behavior of ρ will be relevant in $\mathcal{I}^+$ limit, which is

$$\lim_{t^+\to\infty} e^{-t^+/2}\rho(e^{t^+/2}y) = \Theta(y)\,. \tag{4.20}$$

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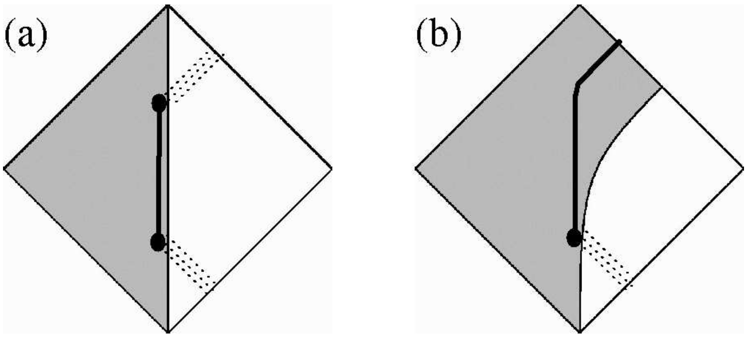

Figure 2: (*a*) In a static background, closed string radiation creates an unstable D0-brane, which subsequently decays back into closed strings. (*b*) Closed string radiation creates a D0-brane which does not decay in time to decay into spacetime modes.

To take the leg pole transform, we want to compute

$$\begin{aligned}\int dv^- \quad dv'^- K(t^- - v^-)K(t'^- - v'^-)e^{(v^- - t^+)/2}\rho(e^{(t^+ - v^-)/2}(1 - \lambda e^{v^-}))\delta(v^- - v'^-) =\\ = \int dv^- K(t^- - v^-)K(t'^- - v^-)e^{(v^- - t^+)/2}\rho(e^{(t^+ - v^-)/2}(1 - \lambda e^{v^-}))\,. \qquad (4.21)\end{aligned}$$

The integral over $v^- > -\ln\lambda$ vanishes in the limit, as can be seen from (4.20). The integral over the rest of v^- gives simply

$$\begin{aligned}\Delta(t^-, t'^-) &= \int_{-\infty}^{-\ln\lambda} dv^- K(t^- - v^-)K(t'^- - v^-)(1 - \lambda e^{v^-})\\ &= \int_{t^- + \ln\lambda}^{\infty} dv^- K(v^-)K(v^- - t^- + t'^-)(1 - \lambda e^{t^- - v^-})\,, \qquad (4.22)\end{aligned}$$

which is obviously the same as (3.53).

Hence, we find that the exact fermion analysis leads to the same conclusions as the collective field analysis concerning the outgoing state on $\mathcal{I}^+$.

5. Nonperturbative effects

In this section, we will, by example, point out why the tachyon theory ceases to be unitary when non-perturbative effects are included. We will use the recently-understood process of D0-brane creation and decay [5, 8] to illustrate the issues.[8]

An unstable D0-brane is represented as a single fermion, corresponding to a single eigenvalue of the matrix model, travelling on a classical trajectory. When it is sufficiently near the Fermi surface, the classical fermion can be rewritten as a perturbation of the Fermi sea, and interpreted as a packet of closed string radiation. Far away from the sea it must have a different description: that of a D0-brane on which open strings can end. Figure 2*a* shows the spacetime process of D0-brane creation and subsequent decay. The same packet of closed string radiation could be travelling on top of our time-dependent

[8]The example we give arose in discussion with J.Polchinski.

Fermi sea. (In the past, the static and time-dependent Fermi seas are essentially identical.) It is possible to choose a set-up in which the single fermion is separated from the Fermi sea in the far future because the Fermi sea drains away before the estranged fermion can reach it. The spacetime picture of this process is shown in figure 2*b*. The D0-brane does not decay — instead, it appears in the no-man's land behind the wall. These nonperturbative degrees of freedom are free to explore the entire coordinate region covered by the $t^{\pm}$ fermion coordinates. This clearly demonstrates that while the tachyon theory as we have described in the collective picture is perturbatively unitary, nonperturbatively it cannot be so. It is missing all degrees of freedom in the no-man's land, and has no hope of describing them since the Fermi surface does not extend into that region.

Of course this is not a crisis for the matrix model because the nonperturbative free fermion theory is perfectly unitary. Nevertheless, there is no unitary spacetime evolution, because spacetime — as described by the fluctuating Fermi surface — does not extend into the no-man's land. This may be an interesting lesson for trying to understand black hole physics in string theory.

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